

- (1) Find the multiplicative inverse for 5 (mod 12).

units: 1, 5, 7, 11

$$5 \cdot 5 = 25 \pmod{12} = 1 \pmod{12}$$

$5 \pmod{12}$

- (2) Find the multiplicative inverse for 2 (mod 21).

units: 1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20

$$2 \cdot 11 = 22 \pmod{21} = 1 \pmod{21}$$

$11 \pmod{21}$

- (3) Does 6(mod 14) have a multiplicative inverse?

$$\gcd(6, 14) = 2$$

6 mod 14 is not a unit

No

- (4) Does 7(mod 15) have a multiplicative inverse?

$$\gcd(7, 15) = 1$$

7 mod 15 is a unit

Yes

- (5) Encrypt the following word using the times cipher  $\star \times \square \pmod{26} = \boxtimes$  given.

- (a) DOG,  $\star = 7$

	D	O	G
$\square = 4$	15	7	
$\star \times 7$	28	105	49
$\boxtimes = 2$	1	1	23
	B	A	W

$BAW$

- (b) TURTLE,  $\star = 11$

	T	U	R	T	L	E
$\square = 20$	21	18	20	12	5	
$\star \times 11$	220	231	198	220	132	55
$\boxtimes = 12$	16	16	12	2	3	
	L	W	P	L	B	C

$LWPLBC$

- (6) You need to decrypt a message using the times cipher  $\star \times \square \pmod{26} = \boxtimes$ , where  $\star = 3$ .

- (a) Find  $\star$ .

$$\text{Since } 3 \cdot 9 = 27 = 1 \pmod{26} \quad \star = 9$$

- (b) Use  $\star$  to decrypt the word FKPPAOE.

	F	K	P	P	A	O	E
$\boxtimes = 6$	11	16	16	1	15	5	
$\star \times 9$	54	99	144	144	9	135	45
$\square = 2$	21	14	14	9	5	19	

$BUNNIES$